Visible effects of the hidden sector

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The renormalization of operators responsible for soft supersymmetry breaking is usually calculated by starting at some high scale and including only visible sector interactions in the evolution equations, while ignoring hidden sector interactions. Here we explain why this is correct only for the most trivial structures in the hidden sector and discuss possible implications. This investigation was prompted by the idea of conformal sequestering. In that framework hidden sector renormalizations by nearly conformal dynamics are critical. In the original models of conformal sequestering it was necessary to impose hidden sector flavor symmetries to achieve the sequestered form. We present models that can evade this requirement and lead to no-scale or anomaly mediated boundary conditions, but the necessary structures do not seem generic. More generally, the ratios of scalar masses to gaugino masses, the μ term, the $B\mu$ term, A terms, and the gravitino mass can be significantly affected.

I. INTRODUCTION: A SIMPLE ARGUMENT FOR CONFORMAL SEQUESTERING

Anomaly mediation [1,2] is generally regarded as an interesting, and possibly predictive, scheme for soft supersymmetry breaking parameters. In order that the anomaly-mediated contributions dominate, however, it is necessary that the Kahler potential has a particular, "sequestered" form. Some arguments for how this structure might emerge for geometrical reasons in higher dimensional constructions were given in Ref. [2]. However, in many situations in string and M theory, where one might have hoped to obtain geometric sequestration, the Kahler potential does not have the required form [3]. The dominance of anomaly mediated contributions in geometric realizations seems to require that the underlying microscopic theory possess (non-Abelian discrete) symmetries acting on the fields of the visible sector. However, such symmetries, if present, can already provide a solution of the supersymmetric flavor problem.

Luty and Sundrum (LS) [4] have proposed an interesting alternative way of realizing anomaly mediation in four-dimensional effective field theories called "conformal sequestering," inspired, in part, by warped constructions in higher dimensions. They argue that in theories in which the hidden sector is conformal over a range of energies, the Kahler potential flows to the sequestered form. In this note, we will review and slightly recast these arguments. We will stress that in the simplest realizations, as noted by LS, additional continuous or discrete symmetries are required to achieve the sequestered form. Amusingly, the symmetries in this case must act on hidden sector fields, as opposed to

visible ones. We will then extend the analysis of LS to a class of models with gauge singlet fields which require smaller continuous or discrete symmetries. We also show that conformal sequestering may be achieved without any hidden sector flavor symmetries, but only under rather exceptional circumstances or with specially chosen couplings to hidden sector singlets.

Apart from the question of anomaly mediation, the observations of Luty and Sundrum lead us to focus on another issue: the role of hidden sector interactions in the renormalization group evolution of operators responsible for visible sector soft-breaking parameters. The standard method to analyze theories with supersymmetry breaking in a hidden sector has been the following. One supposes that the scale of interactions which separates the visible from the hidden sector is M. (We have in mind theories where M is large, such as theories of gravity mediation, or gauge mediation with very massive messengers.) One writes an effective action *for the visible sector fields* at the scale M, treating the hidden sector fields as nondynamical background fields. This is a supersymmetric theory with higher dimensional operators suppressed by the scale M which couple the visible and hidden sectors. One then writes a set of renormalization group equations for the higher-dimensional operators and evolves these into the infrared, *ignoring the hidden sector dynamics*. Supersymmetry breaking F and D components of the background hidden sector fields then give rise to visible sector soft supersymmetry breaking parameters.

Stated in this way, it is clear that if hidden sector fields are treated as dynamical, then in addition to the usual contributions due to visible sector interactions, *hidden sector inter-*

actions can also, in general, give rise to appreciable renormalization group evolution of the operators responsible for soft supersymmetry breaking parameters. By considering the evolution of different types of soft masses, one can quickly derive the results of LS, and extend them to a broader class of models. In this note, we will explain this phenomenon first for globally supersymmetric theories, then for locally supersymmetric theories. We will illustrate the point first using a simple theory where it is easy to perform explicit computations, and we will then discuss gauge theories and conformal field theories, both weakly and strongly coupled. We review the analysis of LS, stressing, as they noted, that discrete symmetries are required in their models if anomaly mediation is to dominate. We then go on to construct models with smaller and possibly no symmetry requirements. The general features of hidden sector renormalization on the operators responsible for direct gaugino masses, the μ and $B\mu$ terms, and A terms are also considered. The classes of models presented here for conformal sequestering that do not require microscopic discrete symmetries are shown to yield either anomaly mediated or no-scale boundary conditions. We conclude with an assessment of anomaly mediation, arguing that, while it might occur in specially chosen models of geometric or conformal sequestering, it does not seem to be a generic outcome of framework more fundamental than that with which we are currently familiar. But more generally, hidden sector interactions can have an important impact on the spectrum of soft supersymmetry breaking parameters.

II. GLOBAL SUPERSYMMETRY

A. Yukawa theories

Consider a theory with two sets of fields, Q and T, with superpotential

$$W = \frac{1}{3!}\lambda T^3, \tag{1}$$

where throughout T are understood to be hidden sector fields while Q are visible sector fields. Suppose that at the high scale, M, there is also a higher-dimension, Kahler potential coupling between the two sectors,

$$\mathcal{L}_{TQ} = c \int d^4\theta\, T^\dagger T Q^\dagger Q, \tag{2}$$

where $c(M) \sim 1/M^2$.

It is straightforward to compute the anomalous dimension of the operator in Eq. (2). In terms of supergraphs, there is only one vertex diagram; one also has to include a wave function renormalization of the T fields. The result is

$$\delta c = 3c \frac{\lambda^2}{16\pi^2}\ln(\mu/M), \tag{3}$$

$$\frac{d}{dt}c = 3\frac{\lambda^2}{16\pi^2}c, \tag{4}$$

where throughout $t \equiv \log \mu$ is the logarithm of the renormalization scale. One can alternatively derive this result using arguments along the lines of Ref. [5]. Think of Q as a background field, and define

$$Z_o = (1 + cQ^\dagger Q). \tag{5}$$

At one loop, Z is corrected; the T propagator is proportional to Z_o^{-1}. So the self energy correction is proportional to Z_o^{-2}, and the T effective Lagrangian is [5]

$$\mathcal{L}_T = Z_o\left(1 - Z_o^{-3}\frac{\lambda^2}{16\pi^2}\ln(\mu/M)\right)T^\dagger T, \tag{6}$$

which yields the same anomalous dimension for the operator $\mathcal{L}_{ttqq}$.

Let us briefly comment on the background field calculation above. Obviously, the wave-function coefficient Z is not a measurable quantity, and physics can not depend on the change of normalization for kinetic terms. Thus we need to specify the effect of the redefinition (5) on the canonical coupling $\lambda_c = \lambda/Z^{3/2}$. To keep the physical coupling unchanged would require a nonholomorphic, field-dependent rescaling of the superpotential coupling λ. Since, in the limit of interest, Q is a light dynamical field, such rescaling is not consistent with maintaining a manifestly supersymmetric Lagrangian. Therefore, the superpotential coupling λ (as opposed to the canonical coupling λ_c) must be held fixed.

Now we can consider the implications of this operator for supersymmetry breaking. Following LS we first imagine that the F component of Q has an expectation value. This gives rise to a soft mass for T,

$$m_t^2 = c|F_Q|^2. \tag{7}$$

According to Eq. (4),

$$\frac{d}{dt}m_t^2 = 3\frac{\lambda^2}{16\pi^2}m_t^2. \tag{8}$$

This agrees with the running of the mass predicted in such a theory by the standard analysis; see, e.g. Ref. [6].

Now we turn to the case of interest and suppose that $F_T \neq 0$ so that, at M,

$$m_q^2 = c|F_o|^2, \tag{9}$$

where $F_o = F_T(M)$ is the F term vacuum expectation value (VEV) of T at the messenger scale. Notice that the soft mass is proportional to the vacuum energy in this globally supersymmetric model. As a result of the running of c, the Q mass evolves. We need to also think about the renormalization of F_T. Generally F_T will be a function of other fields in the model. If we assume that any non-perturbative renormalization of the superpotential is unimportant, then we can write the Lagrangian as before, but $F_T = Z_T^{-1/2}F_o$, where $F_T = F_T(\mu)$ is the F term for the canonically normalized field at the scale μ. Furthermore, F_o should also be written in terms of the canonically normalized fields at μ. So the Q mass renormalizes with both the running of c and the running of

the vacuum energy. Relative to the vacuum energy, running of the soft mass is still given by a factor Z_T^{-3}. If the T sector has a natural scale (e.g., the scale of supersymmetry breaking in the hidden sector, M_{hid}), then the coefficient only runs down to this scale.

To illustrate this, one can take an O'Raiferataigh type model, with

$$W = T(\lambda_o A^2 - \xi_o^2) + m_o BA, \qquad (10)$$

where λ_o, m_o, and ξ_o are "bare" parameters, i.e., parameters of the effective Lagrangian at the scale M. For large m_o, only T develops an F term, $F_{To} = \xi_o^2$. At lower scales, this expression can be written in terms of renormalized fields and parameters, where $\xi^2 = \xi_o^2 / \sqrt{Z_T}$. Now suppose that at the scale M, the theory contains the operator

$$\mathcal{L}_{AQ} = a \frac{1}{M^2} A^\dagger A Q^\dagger Q. \qquad (11)$$

Since there is no F term for A, this operator does not give a soft mass for Q at tree level. However, at one loop, it induces the operator of Eq. (2), with the coefficient,

$$c = 2a \frac{\lambda^2}{16\pi^2} \ln(\mu/M). \qquad (12)$$

(All of the quantities in this expression are renormalized quantities; note that the wave function of T is not important at this order.)

So this example is more dramatic: the soft mass vanishes at the scale M, but it is induced by the low energy dynamics of the hidden sector under renormalization group evolution to lower scales. Again, note that this operator only runs down to the scale of the T interactions (i.e., roughly to the scale ξ).

B. Gauge theories

The effects of hidden sector gauge interactions on the running of operators which couple the visible and hidden sectors can also be included. We wish to consider the renormalization of the operator of Eq. (2). From our analysis of the preceding section, we can proceed very simply. If we suppose first that $F_Q \neq 0$, the operator (2) generates a soft mass for the T scalars. In a pure gauge theory, the running of this soft mass is well known:

$$\frac{d}{dt}(m^2)_j^i = \frac{1}{16\pi^2}\{8\,\delta_j^i g^4 C(i)\mathrm{Tr}[S(r)m^2]\}, \qquad (13)$$

where $C(i)$ is the Casimir invariant appropriate to the scalar in question; the trace is a sum over all of the matter fields, again with $S(r)$ being the appropriate character of the representation.

So we see that if we write the operator of Eq. (2) including the most general flavor structure

$$\mathcal{L}_{ttqq} = \sum_{ij;ab} c_{ij:ab} \int d^4\theta\, T_i^\dagger T_j Q_a^\dagger Q_b, \qquad (14)$$

then, at weak coupling:

$$\frac{d}{dt} c_{ij:ab} = \frac{1}{16\pi^2}\{8\,\delta_{ij} g^4 C(i)\mathrm{Tr}[S(r)c_{ij:ab}]\} \qquad (15)$$

(the trace is on the i,j indices).

This running can be derived more directly working with Feynman graphs. One particularly simple approach is suggested by a discussion of LS. For simplicity, consider a theory with vanishing superpotential and N_f vectorlike flavors. Study the flavor singlet operator:

$$\mathcal{L}_{ttqq} = c \sum_{ij} \int d4\,\theta T_i^\dagger T_j Q_a^\dagger Q_a. \qquad (16)$$

Various components of this operator may be considered. In particular, one of the terms in this expression is a product of currents, $j_T^\mu j_{\mu Q}$. In the limit $c = 0$, these currents are conserved. Since the Q's are treated as noninteracting, it is tempting to say that the operator appearing here is not renormalized. But j_T^μ is in general an anomalous chiral current. If the theory is regulated with a Pauli-Villars regulator of mass M_{pv}, then the renormalization can be determined by noting that because of the conformal anomaly the divergence of the current receives a contribution from the Pauli-Villars fields. The matrix element of the corresponding density $(M_{pv}\bar{\psi}_{pv}\psi_{pv})$ is easily evaluated, and gives Eq. (15).

Note that if we consider an anomaly free hidden sector vector current, there is indeed no renormalization. The same holds for operators which are flavor nonsinglet in visible sector fields (as was discussed in Ref. [4]).

III. LOCAL SUPERSYMMETRY

The analysis of theories with local supersymmetry is similar to the analysis of globally supersymmetric theories described above. We first consider the λT^3 model in the locally supersymmetric case. As an example, we suppose that the Kahler potential is that appropriate to "minimal gravity mediation," i.e., it is universal, with only quadratic terms in the chiral fields in the Einstein frame:

$$K = T^\dagger T + Q^\dagger Q \qquad (17)$$

(where we have restricted our attention to a model with only these two fields). We will ask, to order λ^2, what sorts of corrections are induced to K. Again, we will see that the corrections account for the usual running of the soft masses.

The conventional Einstein frame form for the action is not the most convenient for this analysis. Instead, the standard auxiliary field analysis [7,8] may be done in the superconformal frame, where the Ricci scalar is multiplied by a function Φ, of chiral fields. Φ is related to the Kahler potential by:

$$K = -3 \ln(-\Phi/3). \qquad (18)$$

The Kahler potential of Eq. (17) corresponds to Φ which includes

$$\Phi \supset T^\dagger T + Q^\dagger Q - \frac{1}{6}(T^\dagger T + Q^\dagger Q)^2. \tag{19}$$

In this form, by focusing on particular terms in the component field action, it is easy to compute the renormalization of the various terms. In particular, the term $T^\dagger T Q^\dagger Q$ is renormalized by the same diagrams (in terms of component fields) as in the globally supersymmetric case. In other words, the function Φ, at one loop in hidden sector fields, has the form

$$\Phi \supset (1 + \delta Z)T^\dagger T + Q^\dagger Q - \frac{1}{3}(T^\dagger T Q^\dagger Q)(1 - 2\,\delta Z) + \cdots \tag{20}$$

$$\supset T^\dagger T + Q^\dagger Q - \frac{1}{3}(T^\dagger T Q^\dagger Q)(1 - 3\,\delta Z). \tag{21}$$

where

$$\delta Z = \frac{\lambda^2}{32\pi^2} \ln(\mu^2/M^2), \tag{22}$$

and in the second line we have rescaled (renormalized) the field T. The conventional Kahler potential is now given by

$$K \simeq T^\dagger T + Q^\dagger Q + (1 - 3\,\delta Z)(T^\dagger T Q^\dagger Q). \tag{23}$$

One can also check that this works properly in the Einstein frame. Some of the simplest terms to study are the quartic fermion couplings; it is easy to isolate them, compute their renormalization, and infer the correction to the Kahler potential. If $F_Q \neq 0$, we again reproduce the standard running of the soft mass for T. But again, if $F_T \neq 0$, we see that the soft masses for Q run in the low energy theory.

A. Are hidden sector dynamics important?

In the next section, we will consider the proposal of Luty and Sundrum that the supersymmetry-breaking sector is a strongly coupled field theory, conformal over a range of energies. In this case, the effects of hidden sector renormalization of the quartic terms in the Kahler potential are profound. First, however, we ask what one might expect the size of these effects to be in asymptotically free hidden sector theories which are weakly coupled at the messenger scale. We will see that even in this case they can easily be of order one or larger.

As an example, we take our "hidden sector" theory to be an $SU(N)$ gauge theory with N_f flavors, T_i and $\bar{T}_i$, and a singlet, S. We take the singlet to couple to one of the flavors, with superpotential:

$$W = \lambda S T_1 \bar{T}_1. \tag{24}$$

The one-loop renormalization group equations for this theory are

$$\frac{d\lambda^2}{dt} = -\frac{c\lambda^2}{16\pi^2}g^2 + 2\frac{N+2}{16\pi^2}\lambda^4, \tag{25}$$

$$\frac{dm_T^2}{dt} = \frac{\lambda^2}{16\pi^2}(2m_S^2 + 4Nm_T^2), \tag{26}$$

$$\frac{dg^2}{dt} = -\frac{2b_o}{16\pi^2}g^4, \tag{27}$$

where m^2 are the soft masses squared, $c = 4(N^2-1)/N$, and $b_o = 3N - N_f$. The Yukawa coupling has an attractive quasi-fixed point trajectory given by

$$\frac{\lambda^2}{g^2} = \frac{c - 2b_o}{2(N+2)}. \tag{28}$$

Restricting to this trajectory for simplicity, the one-loop renormalization group equations for m_T^2 are easily solved,

$$m_T^2(\mu) = m_T^2(M)\left(\frac{g^2(\mu)}{g^2(M)}\right)^{[N/(N+2)][(2b_o-c)/b_o]}. \tag{29}$$

As before, we can interpret this as the renormalization of the coefficient of the operator $T^\dagger T Q^\dagger Q$, where Q is a visible sector field, which for the purposes of the calculation above may be thought of as having an auxiliary field expectation value. To get a sense of how large the hidden sector renormalization effects can be, consider the exponent in (29). In the large N limit at fixed N_f it becomes $\frac{2}{3}$. So if the hidden sector gauge coupling becomes strong, the effects can be sizable. The expression (29) is only valid at one loop, but the loop expansion only begins to break down for $g^2(\mu)N \sim 16\pi^2$, illustrating the possibility of very large effects. Strong hidden sector dynamics is common in renormalizable theories of dynamically broken supersymmetry. So large renormalization effects should be typical of a wide classes of models.

Again, there are a variety of ways in which one might want to think about these results. Instead of studying the running of operators due to hidden sector interactions starting at the scale M, one can alternatively (and equivalently) adopt the viewpoint that one should take boundary conditions for the renormalization group equations, not at scale M, but at the lower scale, M_{hid}. Of course, determining these conditions requires knowledge of the microscopic physics at M, and the full renormalization group evolution down to M_{hid}.

B. Gravitino mass

The gravitino mass is of course directly related to the scale of supersymmetry breaking in the hidden sector. Assuming the cosmological constant vanishes by a cancellation between auxiliary and superpotential expectation values, the gravitino mass squared is

$$m_{3/2}^2 = \frac{3|F|^2}{M_p^2}, \tag{30}$$

where here $|F|^2$ is understood to be a sum over all renormalized auxiliary expectation values in the hidden sector. In the notation of Sec. II A, the renormalized values of individual

expectation values are $F = Z^{-1/2} F_o$. So effects of hidden sector renormalization appear also in the gravitino mass.

In many models it is important to consider the effects of hidden sector renormalization on visible sector soft parameters relative to the gravitino mass. This is particularly true for models of conformal sequestering since anomaly mediated contributions to soft masses are suppressed by a loop factor compared with the gravitino mass.

IV. THE LUTY-SUNDRUM MODEL

LS consider the case where the T fields are part of a gauge theory which is nearly conformal over a range of scales. The theories which they consider are required to have a global, non-Abelian hidden sector flavor symmetry. They present a sophisticated argument to determine the running of c, but the main features of the result are clear from our discussion above. Again suppose first, for illustrative purposes, that $F_Q \neq 0$. Then at M, the T fields have a nonzero mass. This will exhibit nearly power-law variation with scale since the hidden sector is nearly conformal. Thus the coefficient c of the operator coupling the visible and hidden sectors does as well. In a strongly coupled theory, the anomalous dimension will be of order 1. Now, as in our perturbative discussion above, we can consider instead $F_T \neq 0$, to argue that the soft masses for the Q's will exhibit power law renormalization since they arise from the same operator.

Actually, the analysis of LS provides further important information: as in our perturbation theory discussion, it demonstrates that hidden sector flavor nonsinglet operators have vanishing anomalous dimensions, while a certain hidden sector flavor-singlet operator has anomalous dimension equal to the first derivative of the beta function at the fixed point [4]. Since, by assumption, the fixed point is attractive, this ensures that the anomalous dimension of this operator is positive. If the coefficients of the nonsinglet operators vanish, then, in the supergravity context, our analysis of the preceding section indicates what happens to Φ: it behaves as

$$\Phi \supset T^\dagger T + Q^\dagger Q + \mathcal{O}(e^{-b't}) T^\dagger T Q^\dagger Q. \tag{31}$$

This is the sequestered form of the Kahler potential. The soft masses arising from direct couplings between the visible and hidden sectors are exponentially suppressed compared with the gravitino mass, and the anomaly-mediated contributions dominate.

In the case of a weakly coupled pure gauge theory (the Banks-Zaks theory [10], to be discussed in more detail below), we can use our earlier arguments to determine the behavior of the various operators. Thinking, again, of the F_Q fields as the origin of soft masses for the T fields, we can use the known results for this evolution. At one loop, the anomalous dimensions of the soft masses (in the absence of gaugino masses and Yukawa couplings) vanish; at two loops they are given by Eq. (13), with g the value of the coupling at the fixed point. Note that one sees immediately that the hidden sector flavor nonsinglet operators (14) have vanishing anomalous dimension. The hidden sector flavor singlet operator (14) with $i = j$ flows to zero in the infrared.

As we will see, even at weak coupling, it is easy to construct conformal field theories with much smaller global symmetries, in which almost all of the operator coefficients tend to zero. This is demonstrated by simply introducing soft masses for the hidden sector fields and studying their evolution. Still, in the examples we study below, it is necessary either to impose some discrete symmetry or arrange rather special couplings in the hidden sector in order that anomaly mediation dominate.

In the Sec. IV A, for completeness, we reproduce the LS argument for the anomalous dimension. In Sec. IV B, we study weakly coupled fixed points, verifying the LS argument, but also developing theories in which all of the soft operators tend to zero.

A. The anomalous dimension of the soft mass

We have given a rather explicit derivation of the running of the soft masses in gauge theory. However, there is another argument which generalizes straightforwardly to strongly coupled theories. The basic strategy, following Arkani-Hamed *et al.* [5], is to think of Z as a vector superfield, whose highest component is the soft mass. This is most easily done for the flavor singlet case. We will return to the nonsinglet case shortly. One wants to determine the Z dependence of the effective action by studying the renormalization group evolution of the lowest component of Z, and then use this to infer the behavior of the highest component. LS consider a situation where $TT^\dagger QQ^\dagger$ should be thought of as giving a correction to Z_T. They then use this sort of argument to infer soft masses for Q. But this can just as well be used to infer soft masses for T (due to $F_Q F_Q^\dagger$).

In a gauge theory, naively, there is only a uniform rescaling of the kinetic terms, i.e., they are always proportional to Z_o, the Z factor at the cutoff scale. This is because factors of Z in vertices are always compensated by factors of Z in propagators. Thinking of Z as including the corrections to the Kahler potential, as in Eq. (5), this would seem to say that the soft masses are not renormalized, since the kinetic terms and soft mass terms would be rescaled by the same factor. On the other hand, we know that at two loops, there is a renormalization of the soft masses.

The resolution of this question was provided by Arkani-Hamed *et al.* As Shifman and co-workers explained originally [9], the Z dependence of the action is more complicated, because introduction of a regulator introduces new Z dependence. This can be described by relating the physical and holomorphic gauge coupling constants through the relation (valid at all scales)

$$\frac{8\pi^2}{g^2(\mu)} = \frac{8\pi^2}{g_h^2(\mu)} - N_c \ln g^2(\mu) - N_f \ln Z(\mu). \tag{32}$$

This relation reconciles the exact Novikov-Shifman-Vainshtein-Zakharov (NSVZ) beta function with the requirement that the coupling constant in the Wilsonian action evolves only at one loop due to the holomorphy. Once wavefunction and holomorphic gauge couplings are promoted to background superfields, the physical coupling must also be

treated as a real superfield. Similarly to the case of the Yukawa theories discussed in Sec. II A, the holomorphic gauge coupling must be held fixed while the perturbation in the Kahler potential is turned on. This requirement together with Eq. (32) determines the relation between the perturbation of $\tau = 8\pi^2/g^2$, and the perturbation of the wave function, δZ:

$$\delta\tau = \frac{\delta\tau_{hol} - N_f \ln(1 + \delta_Z/Z)}{1 - N/\tau}. \tag{33}$$

It is easy to see that $\delta\tau$ does not evolve at one loop.

Let us use this to recover the result for the evolution of the soft masses quoted by Martin and Vaughn [6]. The usual solution of the renormalization group equation for $Z(\mu)$ is

$$Z(\mu) = Z(M)\exp\left(\int_{g(M)}^{g(\mu)} \frac{dg'\,\gamma(g')}{\beta(g')}\right). \tag{34}$$

Writing

$$\beta(g) = -\frac{g^3 b_o}{16\pi^2}, \quad \gamma(g) = c\frac{g^2}{16\pi^2}, \tag{35}$$

this gives

$$Z(\mu) = Z(M)\left(\frac{g_p^{-2}(M)}{g_p^{-2}(\mu)}\right)^{-c/2b_o}. \tag{36}$$

This is the standard evolution of Z in terms of the gauge coupling of the *perturbed* theory, $\tau_p = \tau + \delta\tau$. Similarly to the Yukawa theory, we can expand the above equation in terms of the perturbation $\delta\tau$. The dependence on the perturbation arises at order g^4—as expected, it is a two-loop effect. Expanding the above in $\delta\tau$ and using Eq. (33) gives the correct beta function for the soft mass. This argument is clearly another version of the calculation we described in the preceding section.

B. Weakly coupled conformal field theories

Let us study the problem of soft masses at a Banks-Zaks fixed point. We will start with a conventional analysis, using the formulas in, say, Ref. [6]. The first two beta function coefficients are

$$b_o = -(3N - N_f)\frac{g^3}{16\pi^2}, \tag{37}$$

$$b_1 = -\left[6N^2 - 2NN_f - 4N_f\frac{N^2-1}{2N}\right]\left(\frac{g^2}{16\pi^2}\right)^2 g. \tag{38}$$

Requiring that the beta function vanish gives, to this order,

$$N_f = 3N - \epsilon, \quad \frac{g^{*2}}{16\pi^2} = \frac{\epsilon}{6N^2}. \tag{39}$$

Here we will think of N as extremely large, and of ϵ as an integer of order 1. There will be corrections to this result at higher orders in g. In particular, at higher orders, g^* *is scheme dependent*.

Note that near the fixed point,

$$g = g^* + \delta, \quad \beta = \beta'(g)\delta = b\delta = \frac{\epsilon^2}{3N^2}\delta. \tag{40}$$

So

$$\delta(t) = \delta(0)e^{-bt}. \tag{41}$$

Similarly,

$$\gamma \approx c = -\frac{g^2 N}{16\pi^2} = c + d\delta, \tag{42}$$

where $c = -(N/6)\epsilon$; $d = -(2N/g)(g^2/16\pi^2)$. The renormalization group equation for Z then leads to

$$Z(\mu) = Z(M)e^{ct + (d/b)[\delta(t) - \delta(0)]}. \tag{43}$$

Indeed, for the Banks-Zaks case the LS argument gives the correct anomalous dimension for the soft masses, $\gamma = \beta'$ at the fixed point. The coefficient out front also works out correctly, i.e., one gets the solution with the correct boundary condition to the renormalization group equations.

Following LS, we can obtain a general expression for the anomalous dimension, valid at strong coupling. LS ask how the effective action depends on Z_o. Near the textbook solution of the renormalization group equation to first order in $\delta\tau$, the distance of τ from the fixed point, is

$$Z(t) = [Z(0) + \delta_Z(0)]e^{\gamma^* t + (\gamma'/\beta')[\delta\tau(t) - \delta\tau(0)]}, \tag{44}$$

where γ' and β' are the derivatives of the γ and β functions with respect to τ. From the NSVZ formula for the β function, we also have

$$\beta' = N_f \frac{\gamma'}{(1 - N/\tau)}. \tag{45}$$

Exploiting Eq. (33) we obtain

$$\delta\tau(t) = \frac{\delta\tau_{hol} + N_f \ln[1 + \delta_Z(0)/Z(0)]}{(1 - N/\tau)}e^{\beta' t}. \tag{46}$$

Substituting back into the expression for $Z(t)$, the factor of $\delta_Z(0)/Z(0)$ cancels, and we are left with

$$Z(t) = Z(0)e^{\gamma^* t}e^{\ln[1 + \delta_Z(0)/Z(0)]\delta\tau(t)}, \tag{47}$$

which reproduces the result for the running of the soft mass of the Banks-Zaks fixed point.

C. Purging flavor symmetries

As discussed by Luty and Sundrum, only a particular operator (14) with $i = j$, which is a flavor singlet under hidden sector flavor symmetry, is renormalized in the theory dis-

cussed above. The model has a $U(N_f) \times U(N_f)$ hidden sector flavor symmetry at the classical level. All but one of the operators $T_j T_j^\dagger$ contain, as their $\theta\bar{\theta}$ components, nonanomalous conserved currents, and are thus not renormalized. The one exception is the anomalous axial $U(1)$. As discussed above, it is easy to derive the renormalization of this operator at two loops from the anomaly. If one is to obtain anomaly mediation, it is then necessary to suppress all the operators (14) which are flavor nonsinglet in the hidden sector at the high scale. In Ref. [4], this is achieved through intricate hidden sector discrete symmetries.

But our earlier discussion suggests that if we can construct a hidden sector conformal field theory (CFT) without flavor symmetries, it should be relatively easy to suppress all operators (14) connecting the hidden and visible sectors. As discussed below, it is rather difficult to construct conformal models with only gauge-nonsinglet fields and without continuous non-R global symmetries. But this can be accomplished if hidden sector gauge singlet fields are included with appropriate superpotential couplings. As an example, consider supersymmetric $SU(N)$ QCD with N_f flavors and N colors in the conformal window $\frac{3}{2}N \leqslant N_f < 3N$. We add N_f singlets, S_i, with tree level superpotential

$$W = \lambda \sum_{i=1}^{N_f} S_i T_i \bar{T}_i, \qquad (48)$$

which also results in an interacting conformal theory. To achieve such a coupling, one can impose an S_{N_f} permutation symmetry, or—even better—simply note that the equations for the fixed point have such a symmetry at one loop. Before writing down explicit formulas note that λ has an attractive fixed point; this can be easily verified at weak coupling. As a result, ignoring contributions from hidden sector gaugino masses, many soft masses run to zero (this is a corollary of a theorem of Nelson and Strassler [11]). However, the model has N_f vectorlike conserved $U(1)$'s corresponding to the Cartan subalgebra of the vector $SU(N_f)$ and the overall vector $U(1)$. Correspondingly, these N_f linear combinations of masses (or more precisely, coefficients of quartic operators) do not run.

This models respects an S_{N_f} permutation symmetry at the fixed point, as well as a Z_2 charge conjugation symmetry which exchanges $T_i \leftrightarrow \bar{T}_i$. If we suppose that this discrete $S_{N_f} \times Z_2$ symmetry is exact in the microscopic theory, then the dangerous operators would be absent, and anomaly mediation would dominate. The requisite number of symmetries may be reduced further by introducing additional singlets with different couplings to the quarks. For example, adding $N_f - 1$ additional singlets, S_i', with tree level superpotential

$$W = \lambda' \sum_{i=1}^{N_f-1} S_i' T_i \bar{T}_{i+1} \qquad (49)$$

leaves the overall vector $U(1)$ as the only nonanomalous non-R continuous symmetry. However, for the $SU(N)$ gauge group with hidden sector flavors, the most general singlet

superpotential couplings to mesons which preserve conformal invariance cannot eliminate the overall vector $U(1)$ symmetry. So in this class of models at least an exact discrete Z_2 symmetry in the hidden sector is required in order to suppress all the dangerous operators (14) and obtain conformal sequestering.

The requirement of an exact Z_2 symmetry can be relaxed in special circumstances. For the $SU(N)$ gauge group with flavors this possibility arises only for the special cases of $N = 4$, $N_f = 8$ and $N = 6$, $N_f = 9$. In these cases baryon and antibaryon operators with any flavor structure are exactly marginal and may be added to the superpotential without spoiling conformal invariance. For example, for the $SU(4)$ with $N_f = 8$ theory the superpotential

$$W = \lambda''(T_1 T_2 T_3 T_4 + \bar{T}_1 \bar{T}_2 \bar{T}_3 \bar{T}_4) \qquad (50)$$

breaks the overall vector $U(1)$. With inclusion of a sufficient number of singlets with couplings to mesons such as (48) and (49), all continuous non-R symmetries are then broken. For $SO(N)$ and $SP(N)$ gauge groups with flavors, all the flavor symmetries are axial. So for these two classes of theories with sufficiently many singlets, superpotential couplings of the form (48) and (49) are alone sufficient to eliminate all continuous non-R symmetries without requiring any microscopic discrete symmetries.

In the absence of gauge singlet fields it is rather difficult to construct a single gauge group CFT which does not possess non-R continuous symmetries at the infrared conformal fixed point. A number of requirements must be met to obtain such a theory: First, in the theory with vanishing superpotential there must exist at least one chiral operator which transforms under each nonanomalous continuous non-R symmetry. Second, at least one operator for each symmetry must be exactly marginal with $U(1)_R$ charge $R = 2$. This is a rather nontrivial requirement since it is often the case that operators which break global symmetries are in fact (marginally) irrelevant at a fixed point so that there are enhanced global symmetries in the infrared. And third, each such operator which breaks each symmetry must appear in the superpotential of the interacting theory *after* including the most general set of field redefinitions. Taken together these requirements are rather nontrivial and we expect that they are all satisfied only in rather exceptional circumstances. However, one CFT which does appear to satisfy all these requirements is supersymmetric $SU(4)$ QCD with $N_f = 8$ flavors. As mentioned above the baryon and antibaryon chiral operators of this theory are exactly marginal for any flavor structure, so adding these to the superpotential can break flavor symmetries without spoiling conformal invariance. For example, the superpotential couplings

$$W = \lambda \sum_{i=1}^{8} (T_i T_{i+1} T_{i+2} T_{i+3} + \bar{T}_i \bar{T}_{i+1} \bar{T}_{i+2} \bar{T}_{i+3}) \qquad (51)$$

break the $SU(8) \times SU(8) \times U(1)$ nonanomalous symmetry to $U(1)^{12}$. And the superpotential couplings

$$W = \lambda' \sum_{i=1}^{4} \; (T_i T_{i+2} T_{i+3} T_{i+4} + T_i T_{i+1} T_{i+3} T_{i+4}$$
$$+ T_i T_{i+1} T_{i+2} T_{i+4} + \bar{T}_i \bar{T}_{i+2} \bar{T}_{i+3} \bar{T}_{i+4}$$
$$+ \bar{T}_i \bar{T}_{i+1} \bar{T}_{i+3} \bar{T}_{i+4} + \bar{T}_i \bar{T}_{i+1} \bar{T}_{i+2} \bar{T}_{i+4}) \qquad (52)$$

break these remaining symmetries. So in this theory with at least these superpotential couplings, all the operators (14) are suppressed by hidden sector conformal dynamics and conformal sequestering results without any microscopic discrete symmetries.

We leave to future work the construction of all CFT's without continuous non-R symmetries; but conclude that without rather exceptional circumstances and/or without specially chosen couplings possibly to gauge singlet fields, such CFT's do not appear to be generic.

V. GAUGINO MASSES AND OTHER SOFT PARAMETERS

Hidden sector renormalization can in principle affect any of the operators responsible for visible sector soft supersymmetry breaking parameters, including not only scalar masses but gaugino masses and the μ and $B\mu$ terms. Gaugino Majorana masses require that a gauge singlet field acquire an auxiliary expectation value. This can be the conformal compensator of the supergravity multiplet (as is assumed to dominate in anomaly mediation) or gauge singlet fields directly in the hidden sector. Below we consider the effects of hidden sector renormalization on gaugino mass contributions from the latter type of singlets. This is particularly relevant to the class of nearly conformal models introduced in the Sec. IV C in which hidden sector gauge singlets are introduced in order to relax the requirement of hidden sector flavor symmetries.

The effective operators which yield gaugino masses from direct couplings to the hidden sector are of the form

$$c_\lambda \int d^2\theta \, S W^\alpha W_\alpha, \qquad (53)$$

where S is a hidden sector singlet, W_α is a visible sector superfield strength, and $c_\lambda(M) \sim 1/M$. An auxiliary expectation value $F_S \neq 0$ gives a gaugino mass at renormalization scale μ of

$$m_\lambda(\mu) = c_\lambda(\mu) F_S(\mu). \qquad (54)$$

Since the operator (53) is linear in the singlet, S, the gaugino mass (54) is renormalized by hidden sector dynamics *only* through the wave function of F_S. From the discussion of Sec. II A, the renormalized auxiliary expectation value is $F_S(\mu) = Z_S^{-1/2} F_{S,o}$ where $F_{S,o} = F_S(M)$ and $Z_S(M) \equiv 1$. So the gaugino mass is

$$m_\lambda(\mu) = c_\lambda(\mu) \frac{F_{S,o}}{Z_S^{1/2}}. \qquad (55)$$

Written in this way it is clear that the coefficient c_λ is renormalized only by visible sector dynamics. As an aside, note that unitarity requires the renormalized wave function factor of any gauge singlet field to be greater than one, $Z_S \geqslant 1$. The renormalized auxiliary expectation value appearing in the gaugino mass (55) also appears in the gravitino mass (30) which is likewise renormalized by hidden sector dynamics only through wave function effects. So we see that if supersymmetry breaking is dominated by gauge singlet auxiliary expectation values, then *hidden sector interactions do not modify the operators responsible for direct gaugino soft masses relative to the gravitino mass*. This is in contrast to the operators (2) responsible for scalar soft masses which can be either be suppressed or enhanced by hidden sector renormalization effects, as illustrated in Secs. II A and III A, respectively. This is also in contrast to visible sector renormalization of the operator (53) which is either positive or negative depending on whether the gauge group is asymptotically free or not.

It is also possible to consider the effects of hidden sector renormalization on the operators responsible for the Higgs sector μ and $B\mu$ terms. The superpotential μ term, $W = \mu H_u H_d$, can arise from hidden sector supersymmetry breaking with an operator of the form

$$c' \int d^4\theta \, S^\dagger H_u H_d \qquad (56)$$

where $c'(M) \sim 1/M$. Like the gaugino mass operator (53) this operator is only linear in the hidden sector singlet S. So from the discussion above it follows that if supersymmetry breaking is dominated by gauge singlet auxiliary expectation values then *hidden sector interactions do not modify the operator responsible for the μ term relative to the gravitino mass*. The soft supersymmetry breaking Lagrangian $B\mu$ term, $\mathcal{L} = B\mu H_u H_d$, can arise from an operator of the form

$$c'' \int d^4\theta \, T^\dagger T H_u H_d \qquad (57)$$

where $c''(M) \sim 1/M^2$. This operator is bilinear in fields which either transform under hidden sector gauge interactions or are gauge singlet, and is of the general form of the operators (2) responsible for soft scalar masses. So hidden sector interactions can in principle either suppress or enhance the operators responsible for the direct $B\mu$ term relative to the gravitino mass.

Finally, it is also possible to consider the effects of hidden sector renormalization on the operators responsible for scalar trilinear A terms. These terms arise from operators of the form

$$c_A \int d^4\theta \, S \Phi \Phi \Phi, \qquad (58)$$

where here Φ are understood to be visible sector chiral multiplets and $c_A(M) \sim 1/M$. The specific prefactors of the operators (58) may depend on the underlying theory of flavor. Like the gaugino mass (53) and μ term (56) these operators are also linear in the hidden sector singlet S. So from the

discussions above if follows that if supersymmetry breaking is dominated by gauge singlet auxiliary expectation values, then *hidden sector interactions do not modify the operators responsible for A terms relative to the gravitino mass.*

There are clearly various possibilities for how hidden sector interactions might renormalize the different operators which couple to the visible sector. So small, moderate, or even large ratios of different types of soft supersymmetry breaking parameters could result, depending on details of the hidden sector.

One possibility for large ratios of direct soft parameters to the gravitino mass is the Luty-Sundrum class of models [4]. The hidden sector is assumed to contain only gauge nonsinglet fields and flow to an attractive conformal fixed point. As discussed in Sec. IV C, with appropriately chosen discrete symmetries or a conformal sector without non-R-continuous symmetries, the direct scalar masses are vanishingly small. From the discussion above we see that the direct $B\mu$ term is also vanishingly small. And since there are no hidden sector singlets, the direct gaugino masses, μ term, and A terms also vanish at leading order. Anomaly-mediated contributions to both scalar and gaugino masses then dominate. The μ and $B\mu$ terms must be generated by additional interactions within the visible sector. This class of models, however, suffers the problem of a tachyonic slepton endemic to all models of anomaly-mediated supersymmetry breaking. This unacceptable feature might be cured by yet additional visible sector interactions.

An interesting ratio of direct soft parameters arises for the models of Sec. IV C with a hidden sector with an attractive conformal fixed point, and with sufficiently many singlets with special couplings which break all the continuous non-R-symmetries. In this case, if hidden sector singlets dominate the supersymmetry breaking auxiliary expectation values, then from the discussion above, for general operators coupling the visible and hidden sectors the gaugino masses, μ term, and A terms are of the order of the gravitino mass, $m_\lambda \sim \mu \sim A \sim m_{3/2}$, while the direct scalar masses and $B\mu$ term are vanishingly small, m_ϕ^2, $B\mu \ll m_{3/2}^2$. The leading direct contributions to the scalar masses are anomaly-mediated and therefore loop-suppressed compared with the gaugino masses. So this class of models yields an interesting set of no-scale boundary conditions. Scalar masses are generated mainly from renormalization group effects in the visible sector proportional to gaugino masses and possibly A terms. Aside from the H_u mass, all scalar masses are then nontachyonic. And the $B\mu$ term is likewise generated by visible sector renormalization effects proportional to the product of gaugino masses and μ term. Note that these models yield a phenomenologically viable set of boundary conditions for the correct pattern of electroweak symmetry breaking without postulating additional visible sector interactions to modify soft parameters. However, since the A terms transform under visible sector flavor symmetries, this class of models does not necessarily solve the supersymmetric flavor problem without an underlying theory of flavor.

VI. CONCLUSIONS

From the discussions of the previous sections, we draw two lessons. The first lesson has to do with the idea of con-

formal sequestering. In general, in a conformal field theory, we might expect large corrections to the operators which generate soft masses. LS have provided an elegant analysis which relates the anomalous dimensions of these operators to the derivative of the beta function at the fixed point, in a certain class of theories. But the present analysis also indicates that generic conformal theories possess continuous non-R global symmetries and therefore do not lead, in any general way, to sequestering. Many of the dangerous operators which could violate visible sector flavor symmetries, for example, simply do not run. One must suppose that these operators are absent from the microscopic theory. But in a theory like string theory which contains gravity, there are no continuous global symmetries, so one must suppose that the absence of these terms results from hidden or visible sector discrete symmetries, with a very special structure. Indeed, in the original Luty-Sundrum model, an intricate set of hidden sector discrete symmetries are imposed in order to obtain sequestering. However, we have seen that it is fact possible to obtain conformal theories without non-R continuous symmetries, but only in rather exceptional cases or with specially chosen couplings to gauge singlet fields. In the end, it is not clear to us that this can be viewed, in any sense, as a generic outcome to be expected of some sort of more fundamental theory. Only a full survey of such theories might give an indication.

We view this result as a positive one. In general, in thinking about beyond the standard model physics, there are few limits to our speculations. But the realization that geometrical separation does not lead, generically, to sequestering suggests that anomaly mediation is not a likely outcome of any fundamental theory, such as known string models. Conformal sequestering, as originally proposed, requires the same sorts of discrete symmetries as geometric sequestering, if it is to dominate. We have seen that it may be possible to achieve such sequestering without discrete symmetries, but this phenomenon does not seem generic.

The second lesson is quite general: the evolution of the operators responsible for soft parameters cannot be determined without knowledge of hidden sector dynamics. It is not enough to start with knowledge of the operators which couple the hidden and visible sectors at, say, the Planck scale and evolve these down to low energies using only visible sector dynamics. Hidden sector dynamics will also lead to evolution of the operators. If the hidden sector is, as one might imagine, strongly coupled in order to break supersymmetry, these effects could be quite large. The Luty-Sundrum model is an extreme example of this phenomenon. Alternatively, and equivalently, it is necessary to provide boundary conditions for the renormalization group equations for the operators, not at the scale M, but at the scale at which the hidden sector decouples, M_{hid}.

It should be remarked that the importance of hidden sector dynamics discussed in this paper is an issue for the Wilsonian approach to renormalization, where by definition dynamics in both the hidden and visible sectors are integrated out scale by scale. In this approach the hidden sector can affect physics over all scales down to M_{hid} and must be

taken into account in the running of the nonrenormalizable operators which couple the hidden and visible sectors. However, since the hidden and visible sectors are by definition not coupled through renormalizable interactions, the effects of each sector on the running of nonrenormalizable operators factorize. It is then possible, in principle, to integrate out the hidden sector completely and arrive at a one particle irreducible effective action for visible sector fields alone. In this case, the presence of nontrivial hidden sector dynamics can be subsumed into the boundary conditions for the visible sector soft parameters *at the messenger scale M*. In this approach, hidden sector dynamics does not directly affect the renormalization group running of visible sector parameters below the messenger scale M. But, as discussed here and as is apparent in the Luty-Sundrum model, it can have an important impact on the relative magnitude of different soft parameters at the high scale. In particular ratios of scalar masses to gaugino masses, the μ term, the $B\mu$ term, A terms, and the gravitino mass, can be significantly affected.

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